March 7, 2001 WOCOMAL Varsity Meet

ROUND I: Elementary number theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What is the smallest number that has all of the integers from 2 through 11 as factors?

2. Find the smallest positive integer n such that every digit of 7n is 9.

3. State the greatest common factor in base 12 for 1560_{12} and 210_{12}.

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ____________12__________

Shepherd Hill, Tantasqua, Westborough

March 7, 2001 WOCOMAL Varsity Meet

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $a+2a+3a=8$, what is the value of $3a+4a+5a+6a$?

2. In a group of cows and chickens, the number of legs was 14 more than twice the number of heads. How many cows were there?

3. Solve for x: $\dfrac{1-\dfrac{2}{2-\dfrac{1}{x}}}{1-\dfrac{2}{\dfrac{1}{x}-2}}=\dfrac{1}{4}$

ANSWERS

(1 pt) 1. _______________

(2 pts) 2. _______________

(3 pts) 3. _______________

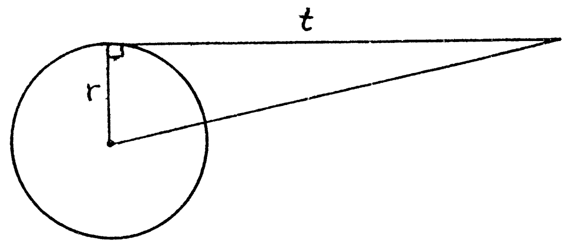
Bromfield, Quaboag, Tantasqua

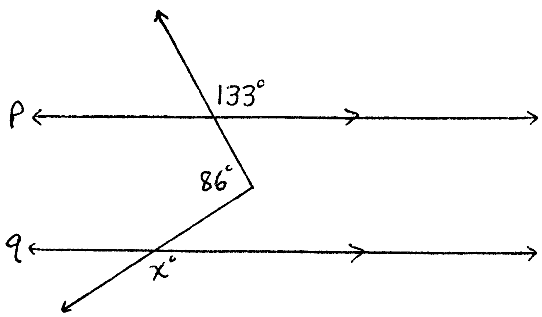
March 7, 2001 WOCOMAL Varsity Meet

ROUND III: Open geometry

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the tangent length t, in terms of the radius r, which will make the area of the right triangle equal the area of the circle.



2. 

In the figure p || q and the angles have degree measures as shown. Find x.

3. In hexagon BDFHJK, $\overline{BD} \perp \overline{BK}$. When $\overline{DF}$ and $\overline{JH}$ are extended, they meet to form an angle of 36°. The interior angles at D, F, J, and K are congruent. Find the number of degrees in angle FHJ.

ANSWERS

(1 pt) 1. t = ________________

(2 pts) 2. ________________

(3 pts) 3. ________________

Hudson, Tahanto, QSC

March 7, 2001 WOCOMAL Varsity Meet

ROUND IV: Logs, exponents, radicals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Simplify: $\dfrac{x^{\frac{1}{2}} + x^{\frac{-1}{2}}}{x^{\frac{-1}{2}}}$

2. Find x if $\log_{10}\left[\log_{x}\left(\log_{5}125\right)\right] = 0$

3. Express as one fraction in simplified radical form: $\dfrac{\sqrt{7}}{\sqrt{5} + \sqrt{2} - \sqrt{7}}$

ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____

Bromfield, Notre Dame, Shrewsbury

March 7, 2001 WOCOMAL Varsity Meet

ROUND V: Trigonometry - open

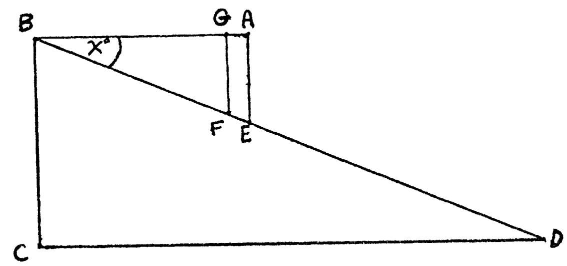
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. For t in radians, find the sum of the amplitude and the period of $y = 2\sin(400\pi t)$.

2. If $(\sin x - \cos x)^2 = a^2$, then express $\dfrac{\sin 2x}{1-a}$ in simplest form in terms of a.

3. $AB = BF = BC = 1$
$\overline{DC} \perp \overline{BC}$, $\overline{AB} \perp \overline{BC}$
$\overline{FG} \perp \overline{AB}$, $\overline{EA} \perp \overline{AB}$

For each trig function name a segment length which equals that function of x.



ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3.

sin x = ________ tan x = ________ sec x = ________

cos x = ________ cot x = ________ csc x = ________

Bartlett, Doherty, Westborough

March 7, 2001 WOCOMAL Varsity Meet

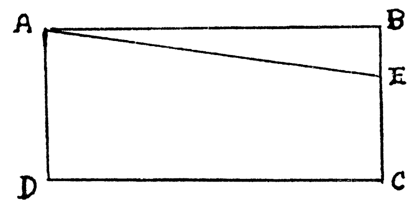
TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1. Determine the number of four digit positive integers which are divisible by 12 and whose digits are all primes.

2. Find the product of the LCM and GCF of $3a^2b^3$, $12ab^3$, and $15a^2bc^3$.

3. Line segment $\overline{AE}$ divides rectangle ABCD into two parts whose areas are in the ratio 6:1. Find the ratio EC:BE.



4. Simplify: $\sqrt[3]{\log_x x^4 \bullet \log_2 16^2 \bullet \frac{1}{\log_{25} 5}}$

5. Solve for θ in degrees, $0^\circ \le \theta^\circ < 360^\circ$: $\sin\theta + \csc^2\theta = 1 - \sin^2\theta + \cot^2\theta$

6. Factor over the reals: $x^2 + 3x + 1$. Hint: This means that radicals may appear in the answer.

7. Write an inequality of the form $|x \pm a| \le or \ge b$ for this graph:

8. Two boats, A and B, leave opposite shores of a lake at the same time and aim at the other boat's starting point. They pass 1000 feet from where A started. Each continues to the opposite shore, immediately turns around and heads back in the opposite direction. They pass the second time 900 feet from where B first started. They travel at constant but different speeds. Asuming no lost time in turning around or passing, what is the distance between their starting points?

9. Find integers a,b,c if $3a - 2b + 4c = 39$, $a + b + c = 50$, and $1 \le a \le 9$.

Assabet Valley, Auburn, Burncoat, South, Uxbridge, QSC

March 7, 2001 WOCOMAL Varsity Meet ANSWERS

ROUND I
thry

1. 1 pt $27,720$

2. 2 pts $142,857$

3. 3 pts 50_{12}

ROUND II
alg 1

1. 1 pt 24

2. 2 pts 7

3. 3 pts $-\frac{3}{4}$ or $-.75$

ROUND III
geom

1. 1 pt $t = 2\pi r$

2. 2 pts 141°

3. 3 pts 78°

degree symbol not needed

ROUND IV
logs
exp
rad

1. 1 pt $x + 1$

2. 2 pts 3

3. 3 pts $\dfrac{5\sqrt{14} + 2\sqrt{35} + 7\sqrt{10}}{20}$

ROUND V
trig

1. 1 pt $2\frac{1}{200}$ or 2.005 or $\frac{401}{200}$

2. 2 pts $1 + a$

3. 3 pts

sin FG tan AE sec BE

cos BG cot CD csc BD

Reverse letter order OK.
Notation $\overline{FG}$ OK.

TEAM ROUND 2 pts each

1. 11

2. $180a^3b^4c^3$

3. $5:2$

4. 4

5. 270°

6. $\left(x + \frac{3+\sqrt{5}}{2}\right)\left(x + \frac{3-\sqrt{5}}{2}\right)$

$\frac{3}{2} + \frac{\sqrt{5}}{2}$ OK

7. $|x+2| \le 6$

8. 2100 ft

9. $a = 5$ $b = 26$ $c = 19$

March 7, 2001 WOCOMAL Varsity Meet BRIEF SOLUTIONS

ROUND I

1 $2\cdot3\ 2\cdot5\cdot7\ 2\ 3\cdot11 = 27{,}720$

2 999 9 ÷ 7 must give an integer. Try 99, then 999, etc until you find that 999,999 ÷ 7 = 142,857

3 $1560_{12} = 1728 + 5\cdot144 + 6\ 12 = 2520$
$= 2^3\ 3^2\ 5\ 7$
$210_{12} = 2\ 144 + 12 = 300 = 2^2\ 3\ 5^2$
GCF $= 2^2\cdot 3\ 5 = 60$
and $60 = 5\cdot12$. Hence 50_{12}

ROUND II

1 $a + 2a + 3a = 6a = 8$
$3a + 4a + 5a + 6a = 18a = 3\cdot6a = 24$

2. Let c = # cows, h = # chickens
$4c + 2h - 14 = 2(c+h)$
$4c = 2c + 14 \Rightarrow c = 7$

3. $$\frac{1-\frac{2}{2-\frac{1}{x}}}{1-\frac{2}{\frac{1}{x}-2}}\cdot\frac{x}{x} = \frac{1}{4}$$ Work on left side for a while

$$\frac{1-\frac{2x}{2x-1}}{1-\frac{2x}{1-2x}} = \frac{2x-1-2x}{2x-1}\cdot\frac{1-2x}{1-2x-2x}$$

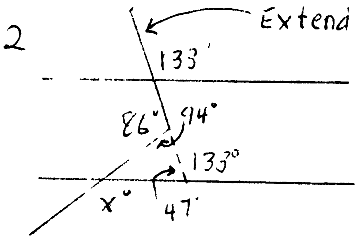
$$= \frac{1}{1-4x} = \frac{1}{4}$$

$4 = 1-4x$, $4x = -3$, $x = -\frac{3}{4}$

ROUND III

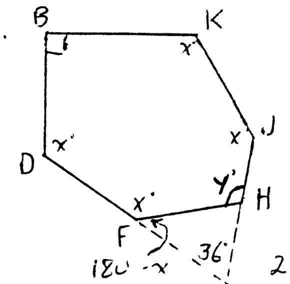
1 rt △ area = circle area
$\frac{1}{2}rt = \pi r^2$
$t = 2\pi r$ (the circumference!)

ROUND III cont

2 

Get other 133° from parallels, 94° and 47° from supplements

Then x is an ext ∠ of a △ which = the sum of the remote int ∠'s, 94° + 47° = 141°

3. 

We want y, ext ∠ of a △.
$y = 180 - x + 36 = 216 - x$
From the hexagon
$y + 4x + 90 = 4\cdot180$
$216 - x + 4x + 90 = 720$
$3x = 414$
$x = 138$
Then $y = 216 - 138 = 78$

ROUND IV

1. $$\frac{x^{\frac12}+x^{-\frac12}}{x^{-\frac12}}\cdot\frac{x^{\frac12}}{x^{\frac12}} = \frac{x^1+x^0}{x^0} = x+1$$

2. $\log_{10}[\log_x(\log_5 125)] = 0$
∴ $\log_x(\log_5 125) = 1$ since $10^1 = 1$
Then $\log_x 3 = 1$ since $5^3 = 125$
and $x = 3$ since $3^1 = 3$

3. $$\frac{\sqrt7}{(\sqrt5+\sqrt2-\sqrt7)}\cdot\frac{(\sqrt5+\sqrt2+\sqrt7)}{(\sqrt5+\sqrt2+\sqrt7)}$$
$$= \frac{\sqrt{35}+\sqrt{14}+7}{5+2\sqrt{10}+2-7}\cdot\frac{\sqrt{10}}{\sqrt{10}}$$
$$= \frac{\sqrt{7\cdot5\cdot5\cdot2}+\sqrt{7\cdot2\cdot5\cdot2}+7\sqrt{10}}{20} = \frac{5\sqrt{14}+2\sqrt{35}+7\sqrt{10}}{20}$$

ROUND V

1 $y = 2\sin(400\pi t)$
amp = 2, period $= \frac{2\pi}{400\pi} = \frac{1}{200}$
sum $= 2\frac{1}{200}$

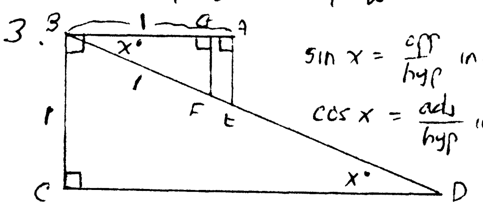
ROUND V cont

2 $\sin^2 x - 2\sin x\cos x + \cos^2 x = a^2$

$1 - \sin 2x = a^2$ or $\sin 2x = 1 - a^2$

Then $\dfrac{\sin 2x}{1-a} = \dfrac{1-a^2}{1-a} = 1+a$

3



$\sin x = \dfrac{opp}{hyp}$ in $\triangle BGF$ = FG

$\cos x = \dfrac{adj}{hyp}$ in $\triangle BGF$ = BG

$\tan x = \dfrac{opp}{adj}$ in $\triangle BAE$ = AE

$\cot x = \dfrac{adj}{opp}$ in $\triangle BCD$ = CD

$\sec x = \dfrac{hyp}{adj}$ in $\triangle BAE$ = BE

$\csc x = \dfrac{hyp}{opp}$ in $\triangle BCD$ = BD

denom = 1 in each case

TEAM ROUND

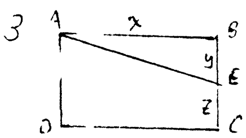
1 Prime digits are 2, 3, 5, 7. Need divisibility by 4, so last 2 digits are 32, 52, or 72. Also need divisibility by 3 so digit sum must be divisible by 3. They are 2232, 2532, 2772, 3372, 3732, 5232, 5532, 5772, 7272, 7332, 7572, 2352, 3252, 3552, 5352, 7752 — 16 of them

2 $3a^2b^3,\ 2^2 3ab^3,\ 3\cdot 5a^2bc^3$

LCM $= 2^2 3\cdot 5a^2b^3c^3$ and GCF $= 3ab$

Product $= 180a^3b^4c^3$

3



Want $\dfrac{z}{y}$

$\dfrac{\text{area } \triangle ABE}{\text{area rect } ABCD} = \dfrac{1}{7}$

$\dfrac{\frac{1}{2}xy}{x(y+z)} = \dfrac{1}{7}$, $\dfrac{y}{2y+2z} = \dfrac{1}{7}$

$7y = 2y + 2z$, $5y = 2z \Rightarrow \dfrac{z}{y} = \dfrac{5}{2}$

4 $\sqrt[3]{\log_x x^4 \cdot \log_2 16^2 \cdot \dfrac{1}{\log_{25} 5}}$

$= \sqrt[3]{4 \cdot \log_2 2^8 \cdot \dfrac{1}{\frac{1}{2}}}$

$= \sqrt[3]{4 \cdot 8 \cdot 2} = \sqrt[3]{64} = 4$

TEAM ROUND cont

5 $\sin\theta + \csc^2\theta = 1 - \sin^2\theta + \cot^2\theta$

$\sin\theta + \csc^2\theta = \csc^2\theta - \sin^2\theta$

$\sin\theta + \sin^2\theta = 0$

$\sin\theta(1 + \sin\theta) = 0$

$\sin\theta = 0$ reject because of $\csc\theta$ $\sin\theta = -1$, $\theta = 270$

6 Solve $x^2 + 3x + 1 = 0$ by the quadratic formula. Get roots r_1 and r_2. Factoring is $(x - r_1)(x - r_2)$. $x = \dfrac{-3 \pm \sqrt{9-4}}{2}$

$\left(x + \dfrac{3-\sqrt5}{2}\right)\left(x + \dfrac{3+\sqrt5}{2}\right)$

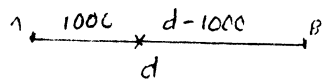
7 Interval length = 12, midpt = −2

Distance between x and −2 ≤ 6

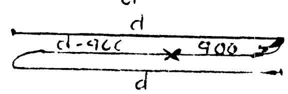
$|x+2| \le 6$

8 Let d = desired distance

1st passing



2nd passing



Since the speeds are constant, the ratio $\dfrac{\text{dist to 1st pass}}{\text{dist to 2nd pass}}$ is the same for both

$\therefore \dfrac{1000}{d+900} = \dfrac{d-1000}{2d-900}$

$2000d - 900000 = d^2 - 100d - 900000$

$0 = d^2 - 2100d$ and $d > 0$

$d = 2100$ ft

9 2nd eq $4a + 4b + 4c = 200$

1st eq $3a - 2b + 4c = 39$

subtr $a + 6b = 161$

$a = 161 - 6b$

Since $1 \le a \le 9$, $1 \le 161 - 6b \le 9$

or $-160 \le -6b \le -152$

$\therefore 26\frac{2}{3} \ge b \ge 25\frac{1}{3}$, but b = integer = 26

Then $a = 161 - 6\cdot 26 = 5$

and $c = 50 - a - b = 19$